Title
A Robotic Intelligent Towing Tank for Learning Complex Fluid-Structure Dynamics

Short titles
Intelligent Towing Tank

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Abstract
We describe the development of the Intelligent Towing Tank, an automated experimental facility guided by active learning to conduct a sequence of vortex-induced vibration (VIV) experiments, wherein the parameters of each next experiment are selected by minimizing suitable acquisition functions of quantified uncertainties. This constitutes a paradigm shift in conducting experimental research, where robots, computers and humans collaborate to accelerate discovery and search expeditiously and effectively large parametric spaces, impracticable with the traditional approach of sequential hypothesis-testing and subsequent train-and-error execution. We describe how our research parallels efforts in other fields, providing an impressive orders-of-magnitude reduction in the number of experiments required to explore and map the complex hydrodynamic mechanisms governing the fluid-elastic instabilities and resulting nonlinear VIV responses. We show the effectiveness of the methodology of “explore-and-exploit” in parametric spaces of high dimensions, which is intractable with traditional approaches of systematic parametric variation in experimentation. We envision this active learning approach to experimental research can be used across disciplines and potentially lead to new physical insights and a new generation of models in multi-input/multi-output nonlinear systems.
Summary

ITT studied VIV in a large parametric space, showing how active-learning experimentation can accelerate scientific discovery.

MAIN TEXT

Introduction

It is 3:00 AM in mid-winter in Cambridge, Massachusetts. The MIT Sea Grant Hydrodynamics Laboratory is pitch-black and vacant, but a periodic sound comes from the Intelligent Towing Tank (ITT) every 2-4 minutes, lasting one minute. ITT works continuously day and night without any interruption or supervision.

In this paper, we describe the new robotic ITT, which we began operating only last year at the Massachusetts Institute of Technology. The ITT has already conducted about 100,000 experiments, essentially completing the equivalent of all of a PhD student’s experiments every two weeks. Indeed, over the last 30 years at a similar MIT lab, a typical doctoral student would finish her PhD in about five years, having completed no more than a thousand laborious experiments (1).

The total number of experiments completed by the ITT in its first year of operation is perhaps comparable to all experiments done collectively to date by all of the different labs in the world on the subject of vortex-induced vibrations (VIV). But more than that, by deploying the Gaussian process regression (GPR) as the learning algorithm in the ITT, we experimentally studied VIV problems within a much wider parametric input space than ever explored before. In doing so, we demonstrated a potential paradigm shift in conducting experimental research, where computers (advancement in AI technology (2)), robots (increasing use of laboratory automation (3-5)) and humans can collaborate in real time to accelerate scientific discovery. In such a shift, robots, computers and humans may search expeditiously and effectively to explore very large parametric spaces, not possible with the traditional approach of sequential hypothesis-testing and train-and-error execution.

Our laboratory is not the only one utilizing this symbiosis between research, machines and science. At Carnegie Mellon University, researchers are training robots to conduct chemical work. The “robot researcher” will decide how to modify substances and reactions without the need for human intervention (6). A number of similar tools have also been developed and used in the life sciences (7), e.g., involving an intelligent Robot Scientist, “Adam”, whose role is to generate new hypotheses for functional genomics and test them (8, 9). Another Robot Scientist, “Eve”, has reportedly tested drugs successfully for malaria (10). These “robot scientists” are similar to what we have achieved with the ITT, where the computer decides which combination of parameters (e.g., speed, amplitude, and frequency) will be investigated and executed next by robots based on the targeted quantity-of-interest (QoI).

A further development along these lines of revolutionizing scientific research is provided by the Big Mechanism program of DARPA (11). The computer reads tens of thousands of papers and synthesizes a hypothesis that can then be tested in a lab by humans, or robots, or a combination of all three – computers, robots, and humans. A vivid manifestation of this vision, at least in part of the upstream process, is provided by the recent publication of a book authored by the “Beta Writer” on a machine-generated summary of current research on lithium-ion batteries (12). The Big
Mechanism of DARPA is another upstream component of the entire robot-computer-human research process to accelerate scientific inquiry and discovery. In the current paper, however, we are focusing on the formulation of the hypothesis-testing, and particularly on its effective execution, i.e., a downstream process.

Fifty years ago, this type of paradigm shift and potential revolution in scientific research and beyond was envisioned by the interactive computing pioneer J.C.R. Licklider, who hoped that, “in not too many years, human brains and computing machines will be coupled together very tightly, and that the resulting partnership will think as no human brain has ever thought and process data in a way not approached by the information-handling machines we know” (13). Today, twenty years into the twenty-first century, we see such a vision being finally realized with different scientific fields contributing creatively to this metamorphosis of the scientific inquiry and discovery.

In the following sections, we first describe the problem to study, provide an overview of our approach, and present the results. Then, we discuss conclusions and limitations of the current research and main future directions. The Materials and Methods section reviews the GPR learning algorithm used in our facility and defines the hydrodynamic quantities studied in the paper.

**Problem description:** The field of fluid-structure interactions is very rich in physical complexity both in hydrodynamic and aerodynamic flows, and one of the canonical problems in the field is VIV (14, 15). VIV occurs when a flexibly-mounted bluff body is placed within an oncoming cross-stream that a spontaneous instability in the wake of the body, above a Reynolds number (Re) of about Re = 50, causes the formation of asymmetric vortical patterns, which induce unsteady loads on the body and hence a vibratory response (16). Over a hundred years ago, Strouhal made a major experimental contribution when he showed that for subcritical Re, alternating sign vortices form behind a circular cylinder at a distinct non-dimensional frequency \( fd / U \) of about 0.20 (17), where \( d \) is the diameter of the cylinder, \( f \) is the frequency of vortex formation and \( U \) is the stream velocity. When the cylinder is flexibly mounted, it vibrates harmonically but at a frequency somewhat different than the Strouhal frequency, influenced by the natural frequency of the structure, and the effective added mass of the cylinder, which varies with frequency.

The complexity of the physical mechanisms that lead to the vibrations of the structure make prediction very difficult. Bishop and Hassan made the successful hypothesis that when a cylinder is forced to vibrate in the cross-flow direction at the frequency and amplitude of a freely vibrating cylinder, it would be subjected to identical forces (18). Under the assumption that free vibrations are steady-state and harmonic, exploring the fluid force dependence in forced vibrations as a function of the principal parameters – viz. the amplitude and frequency of oscillation – would allow the prediction of free vibrations as well. As a result, several studies (1, 19-22) followed that mapped the force dependence and made the connection of the force parametric dependence with changes in the vortical patterns (23, 24).

Hence, using force vibration data was shown to be a powerful tool to predict VIV. Re was found to influence the vibrational properties, leading to higher amplitudes of free vibrations in the subcritical regime with increasing Re (25), and then sharp changes within the critical Re range, with a return to smooth amplitude variations in the supercritical regime (26, 27); therefore,
extensive forced vibration testing as a function of $Re$ was required (28). The discovery that in-line motions affect cross-flow vibrations significantly (29) added further parametric complexity, requiring inclusion of the amplitude of in-line vibration and the phase angle between in-line and cross-flow response. Also, long flexible structures placed in shear currents, such as cables and risers, are subject to multi-frequency vortex-induced responses (30, 31); it is fortuitous that strip theory is found to be largely valid; hence, again, forced vibrations can be used to explore the properties of VIV.

In summary, forced vibrations have become a uniquely effective tool in exploring the very complex properties of VIV (32), leading to the discovery of important properties and the development of comprehensive databases; however the number of independent parameters required to predict the vibratory response of flexible structures in sheared flows is large, making a systematic parametric search intractable, i.e., assuming that we typically have a 10-dimensional parametric space and we blindly conduct 10 measurements per parameter, this brute-force approach would require 10 billion experiments, which is clearly infeasible. Therefore, we hereby introduce active learning to endow the ITT with intelligence to automatically conduct a sequence of forced vibration experiments to study VIV, wherein the parameters of the next experiment to conduct are selected by minimizing suitable acquisition functions (33). In this way, we show that we reduce the experimental burden by several orders of magnitude, requiring only a few thousand experiments, while the choice of each next experiment is made by the computer, as we describe in detail in the next section, thus automating and accelerating the experimental effort.

**Approach overview:** The experimental facility constructed for our study is the ITT shown in Fig. 1. It consists of a towing tank, a robot, and a computer. The ITT has a towing length of 10m and a 1m$\times$1m test cross-section. The main carriage is installed on two rails aligned with the tank length and is able to reach a constant velocity from 0.01m/s to 1.50m/s. On the carriage, a stage is installed with three degrees of freedom, allowing combined trajectories of in-line (align with the towing direction), cross-flow (perpendicular to the towing direction) and rotation motions. The software of the experimental facility is developed with integrated capability of the motion update and trajectory monitoring (PowerPMAC system), force measurement (NI DAQ-USB6218 with an ATI-Gamma 6-axis force sensor) and GPR (34) learning in MATLAB.

Using disciplinary knowledge, we first identify the input parameters and their ranges that may affect the QoIs and pass this information to the ITT. In the future, this first step too could also be automated using approaches like the aforementioned “Big Mechanism”. Next we can start exploration and exploitation of the parametric space adaptively and in sequence, and automatically perform the corresponding experiments to predict QoIs. The flowchart in Fig. 1 presents the main steps of the adaptive sequential experimentation by the ITT. The process starts with a small number of experiments with inputs randomly selected in the parametric space (the initial number of tests has to be larger than the number of the parameters.). After the new experiment, the ITT performs learning with GPR on the existing data to update its prediction on QoIs (a brief overview of GPR is provided in Material and Methods). Meanwhile, we find the inputs of the next experiment by minimizing the acquisition function, which describes the uncertainty through the standard deviation as a function of the parameters (35). Before running the next experiment, a pause period is enforced to quiet down the fluid motion and avoid cross-contamination of the results. Then after the new data has been gathered, the next iteration of learning process begins.
The learning stops when the prediction of the QoIs is converged. Here we track the maximum of the standard deviation $\sigma_{\text{max}}$ in the iteration to be stably smaller than a reference level. As GPR learning is a stochastic process, such a reference of the convergence should be associated with the inherent system uncertainty. We classify the system uncertainty into two types due to (a) modeling and (b) measurement. The modeling uncertainty arises from the selection of the surrogate model and the optimization methods for the learning, while the measurement uncertainty comes from the sensor noise as well as the physical uncertainty associated with the problem, e.g., background turbulence in the water. Ideally, if we can map the unknown function of the QoI perfectly, there will be a zero modeling error, and hence, the predicted uncertainty will converge to the measurement uncertainty. The measurement uncertainty is an inherent property of the experimental facilities (varies by facility) and has to be calibrated beforehand.

In our VIV studies, we quantified the measurement uncertainty of the ITT by evaluating the standard deviation of a baseline case: repeated experiments for the mean drag coefficient $C_d$ of a stationary circular cylinder in uniform flow at $Re = 12,000$ (the mean and standard deviation of the results are provided in the Supplementary Material – Table S1 and Fig. S1.). The mean and the standard deviation of $C_d$ are found to be equal to 1.198 and 0.0398, respectively. We must point out that the variation of $C_d$ originates not only from sensor noise but also from the three-dimensional nature of the vortex shedding affecting the correlation length of the vortical structures in the wake of a stationary circular cylinder, whose length varies in the range of 3 to 5 diameters (36). In our experiment, our cylinder length-to-diameter aspect ratio is 12.3, and hence the vortex shedding process is not fully correlated along the entire model span. This result is comparable with previous literature (1) of $Re = 10,000$ conducted in a different facility. We choose to multiply this baseline standard deviation with a factor to define the convergence reference level for the measurement uncertainty of all QoIs in our study. Taken together with the physical arguments and the inherent uncertainty of the facility, the prediction of QoIs is considered converged and therefore the ITT learning stops when $\sigma_{\text{max}}$ of 10 successive iterations is found to be smaller than $3\sigma_r$.

Typical results of the GPR learning process described above are shown in Fig. 2, where the QoI is the lift coefficient in-phase with the velocity $C_{lv}$ of a cross-flow only forced vibrating rigid cylinder in uniform flow at Reynolds number $Re = 12,000$. The definitions of the hydrodynamic coefficients are given in the section Materials and Methods section. In Fig. 2, the arrows indicate the GPR learning sequence. With 15 experiments (Fig. 2 (A.1), the black dots denote the data used in the learning for the contour), the ITT finds that $C_{lv}$ can be both positive and negative, separated by the red contour line of $C_{lv} = 0$. More importantly, the quantified uncertainty guides the next experiment at $f_r = 0.1455$ and $A_r/d = 0.4571$ where the $\sigma$ is found as the maximum $\sigma_{\text{max}}$ in the standard deviation plot (inset). After the new data has been gathered, the ITT updates both its prediction and the quantified uncertainty of the QoI. The ITT observes a larger positive region for $C_{lv}$ in Fig. 2 (A.2), while the next experiment selected will be performed at $f_r = 0.1427$ and $A_r/d = 1.35$. With the increase in the number of experiments, the ITT reveals more details about $C_{lv}$ versus $f_r$ and $A_r/d$. In the meantime, the value of $\sigma_{\text{max}}$ found in each iteration in Fig. 2 (B) is shown to decrease approaching the $3\sigma_r$ reference line. Between Fig. 2...
(A.3) of 36 and (A.4) of 37 experiments, a new feature of a second positive region of $C_{lv}$ emerges, accompanied with a slight increase of $\sigma_{\text{max}}$ shown in Fig. 2 (B). Eventually, upon convergence, the ITT has learned the $C_{lv}$ pattern, whose contours do not change by performing additional experiments. Fig. 2 (A.5) selects the case of 80 experiments as a representative while the entire evolution of the 176 experiments can be found in the Supplementary Material - Fig. S2. The process of the ITT sequential experimentation has been recorded and documented in the Supplementary Material – Video S1.

**Kernel selection:** The GPR learning performance, viz. the convergence rate, depends on the selection of the surrogate model (see an overview of GPR in the Material and Methods). Fig. 3A, B, C plot the $\sigma_{\text{max}}$ over 200 iterations of GPR learning QoI with different kernel functions for $C_d$, $C_{lv}$ and $C_{my}$ respectively, of a rigid cylinder forced vibration in uniform flow at $Re = 12,000$. We see that in comparing the performance among different kernel functions with a fixed basis function, the learning processes for all three hydrodynamic coefficients converge eventually, but with different rates of convergence. After the test of the different basis and kernel function combinations, based on the convergence rate, we discovered the best combinations for the different hydrodynamic coefficients and used them for the rest of our study, listed in the Table 1.

**Results**

Next, we demonstrate how GPR learning and the ITT accelerate the route to discovery by comparing with the traditional approach of manual uniform sampling of parametric space. We also demonstrate how the ITT allows us to explore wider parametric spaces for possible discovery of new insights and universal scalings.

**GPR adaptive learning versus uniform sampling:** The ITT first learns the three hydrodynamic coefficients ($C_d$, $C_{lv}$ and $C_{my}$) versus $f_r$ and $A_y/d$ of a rigid cylinder in cross-flow only forced vibration and uniform flow at $Re = 12,000$ using a multi-output GPR learning strategy. The results are compared with the reference experiment using uniform sampling on a lattice, which includes 2,268 experiments with 28 different values of $f_r$, 27 different values of $A_y/d$ and three repeated runs for each $f_r$ and $A_y/d$ combination. The multi-output GPR learning process updates the prediction of the multiple QoIs in batches, as between two iterations, multiple experiments are conducted based on the searching of $\sigma_{\text{max}}$ for each QoI. The learning of a QoI stops when the convergence criterion is met, and the whole process stops when all QoIs have converged based on the aforementioned criteria.

The comparison of the results is shown in Fig. 4 for $C_d$ (first row, 75 experiments for GPR learning), $C_{lv}$ (second row, 77 experiments for GPR learning) and $C_{my}$ (third row, 90 experiments for GPR learning). To quantify the difference between the two sets of experiments, we selected 30 points in the parametric space, denoted as the blue dots in Fig. 4 (A.2), to calculate their average value from the reference experiment, as well as the average of the

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prediction and standard deviation at those 30 points for each iteration of the GPR learning sequential experiments, as follows,

$$\bar{C} = \sum_{i=1}^{30} C_i, \quad \sigma = \sum_{i=1}^{30} \sigma_i.$$  \hspace{1cm} (1)

The results are plotted in Fig. 4 for $C_d$ (A.3), $C_{lv}$ (B.3), and $C_{my}$ (C.3). We see that with the increase in the number of experiments, the difference of the prediction average of the corresponding 30 points between the reference experiments (red dashed line) and the GPR learning experiment (black dashed line) in each iteration becomes smaller, and the $2\sigma$ margin (in blue shade) decreases. Furthermore, such a difference between the average value of the reference and GPR learning experiments is always in the $2\sigma$ error margin (representing 95% confidence).

Based on this comparison of the experiments using adaptive GPR learning and uniform sampling, we see that with a less than 5% of the total reference experimental runs, the ITT using the GPR learning strategy is able to capture the major features of the hydrodynamic coefficients of a rigid cylinder in uniform flow at $Re = 12,000$, as follows: (a) $C_d$ is found to vary from 1 to 4 and increase with $f_r$ and $A_y/d$; (b) $C_{lv}$ has two positive regions in the parametric space; (c) $C_{my}$ is found to change drastically from a negative to a large positive value around $f_r = 0.16$.

Previous research has shown – using flow visualization – that such changes are due to a wake mode change from the so-called “2P” pattern to “2S” pattern (37).

**Reynolds number effect:** The effects of the physical nonlinearities become stronger with $Re$ for VIV, so next we focus on learning the $Re$ effect of an oscillating cylinder in the cross-flow direction. Previous research has shown that $Re$ has played a significant role in the Strouhal number (St) (non-dimensional frequency) and the mean drag coefficient $C_d$ of a stationary smooth circular cylinder (38, 39). However, only limited work has been performed on an oscillating cylinder at various $Re$ values, since adding $Re$ as a parameter substantially increases the number of experiments required, and hence the complexity of the experimental problem. Nevertheless, limited available studies show that even within the subcritical regime, $Re$ plays an important role in affecting the fluid forces and the wake states (24). Hence, for the second set of experiments, in addition to $f_r$ and $A_y/d$, we include $Re$ as the third input parameter, ranging from $Re = 1,200$ to $Re = 19,000$ of the subcritical regime.

Fig. 5 plots the converged hydrodynamic coefficients (A: $C_d$, B: $C_{lv}$, and C: $C_{my}$) in the three-dimensional parametric space ($f_r$, $A_y/d$, $Re$) from $Re = 1,200$ to $Re = 19,000$. We find that: (a) $C_d$ does not depend strongly on $Re$; (b) at higher $Re$, the iso-surface of $C_{my} = 0$ depends only on $f_r$ while at lower $Re$ the iso-surface of $C_{my} = 0$ is also a function of $A_y/d$; (c) $C_{lv}$ has two separate positive regions consistently over the entire range of $Re$ we studied; and (d) $C_{lv}$ describes the average energy transfer over time between the fluid and structure. Therefore, if the structure has zero damping, when the flexibly-mounted cylinder has reached a steady-state vibration, the fluid energy flux will be zero, and hence $C_{lv}$ will be equal to zero. Fig. 5 (B) shows
that with increasing $Re$, the maximum $A_y/d$ associated with the $C_{lv} = 0$ iso-surface increases from $A_y/d = 0.75$ to $A_y/d = 1.15$ over the studied $Re$ range (see also the Supplementary Material – Fig. S3 where we show contours of $C_{lv}$ from GPR learning at various $Re$ number values, and contours of $C_d$ and $C_{my}$ at various $Re$ number values are shown in the Supplementary Material – Fig. S4 and Fig. S5). This results in an increasing amplitude response of the cylinder cross-flow only free vibration in the subcritical $Re$ range when $Re$ increases from 1,200 to 19,000, which has, indeed, been reported in previous work (25). This shows how the new physics discovered in forced vibrations with the ITT can explain established results in literature for free vibrations.

It should be noted that our experimental facility is typical of laboratory-size facilities that target subcritical $Re$. Testing at high $Re$ requires large facilities, where cost and time required to conduct them rise as the third power of $Re$; our methodologies hold even greater promise for testing at high $Re$, as few experiments are possible to cover a wide parametric range.

**Larger parametric space:** Using the assumption of strip theory, the hydrodynamic coefficients acquired from the rigid cylinder forced vibration experiment can be used to predict the VIV of a marine riser placed in sheared ocean current profiles (40). However, the riser response is not limited to cross-flow vibrations at a single frequency only; it involves an in-line response as well, which is coupled to the cross-flow motion, and, in addition, multiple frequencies may be excited (30, 31). Hence in the third task, the ITT aims to learn a single QoI of $C_{lv}$ for a rigid cylinder undergoing combined inline and cross-flow forced vibration in uniform flow at $Re = 5,715$ and at either a single frequency or two, as follows,

$$
(C_{lv})_{\text{single}} = C_1 \left( \frac{A_y}{d}, \frac{A_x}{d}, V_r, \theta \right),
$$

$$
(C_{lv})_{\text{double}} = C_2 \left( \left( \frac{A_y}{d}, \frac{A_x}{d}, V_r, \theta \right), \left( \frac{A_{y2}}{d}, \frac{A_{x2}}{d}, V_{r2}, \theta_2 \right) \right),
$$

where the ranges of $A_y/d$ and $A_{y2}/d$ are selected within $[0.05, 1.2]$; $A_x/d$ and $A_{x2}/d$ are selected within $[0.05, 0.4]$; $\theta$ and $\theta_2$ are selected in $[0, 2\pi]$; $V_r = 1/f_r$ is the reduced velocity, the inverse of the reduced frequency and is selected in $[4, 8]$ and $V_{r2}$ is selected in $[2, 15]$. This part is also an important step in parametric analysis and is based on disciplinary knowledge but future work could also automate this part using the aforementioned concepts for the upstream preparation for experimentation to formulate proper hypotheses and scalings (11-13).

Compared to approximately $10^8$ number of experiments required for double frequency tests using uniform sampling strategy, the ITT obtains converged results of $C_{lv}$ with 3,944 experiments. In order to show the effect of the second vibration frequency on $C_{lv}$ associated with the first frequency, we define $\chi'$ to be the average value of $C_{lv}$ for the double frequency experiment
with input \( \left( \frac{A_y}{d}, \frac{A_x}{d}, V_r, \theta \right)^i, \left( \frac{A_y}{d}, \frac{A_x}{d}, V_{r2}, \theta_2 \right)^i \) that 2,400 \( \left( \frac{A_y}{d}, \frac{A_x}{d}, V_r, \theta \right) \) combinations are randomly selected in the parametric space:

\[
\chi' \left( \frac{A_y}{d}, \frac{A_x}{d}, V_r, \theta \right) = \frac{1}{N} \sum_{i=1}^{N} \left( C_{iv} \right)_{\text{double}}^{i,j} = \frac{1}{N} \sum_{i=1}^{N} C_2 \left( \left( \frac{A_y}{d}, \frac{A_x}{d}, V_r, \theta \right)^i, \left( \frac{A_y}{d}, \frac{A_x}{d}, V_{r2}, \theta_2 \right)^i \right),
\]

where \( N = 2,400 \) and standard Morris sensitivity analysis (41) is performed on \( \chi' \), with 100 discrete levels along each dimension of the parametric space and 1,000 elementary effects per parameters, resulting in \( j = 1, 2, \ldots 5,000 \).

Fig. 6 plots the comparison of \( C_{iv} \) versus \( V_r \) and \( \theta/\pi \) for rigid cylinder inline-and-cross-flow-combined forced vibration in uniform flow at \( Re = 5,715 \) between single and double frequency experiments at fixed \( A_y/d = 0.15 \) and \( A_x/d = 0.75 \), as well as the sensitivity analysis on \( \chi' \).

The results reveal that: (a) with the existence of the second frequency component, \( C_{iv} \) associated with the first frequency is found to be dependent on \( \theta \). More specifically, positive \( C_{iv} \) is found to be mainly associated with \( \theta \in [0, \pi] \) of counter-clockwise inline and cross-flow trajectory, similar to that of the single frequency vibration (26, 29), shown in Fig. 6 (A); (b) the sensitivity analysis in Fig. 6 (C) indicates that \( V_{r2} \) has a much stronger effect on \( C_{iv} \) associated with the first frequency compared to \( A_y/d, A_x/d \) and \( \theta_2 \). where \( \mu_x \) on the x-axis is the mean of the individual elementary effects (thus, the sensitivity of the parameter tested alone). Also, \( \sigma_x \) on the y-axis represents the standard deviation of the elementary effects (thus, the sensitivity of the parameter tested in interaction with other parameters), which is also revealed by comparison among B.1 to B.5.

**Discussion**

Our results bear great promise for accelerating discovery in experimental science and for a potential paradigm shift in experimental labs around the world for new research procedures based on a combination of robots, machine learning and humans synergistically. The idea is simple to implement as we resort to laboratory robots to perform automatic sequential learning tasks of studying the scientific hypothesis raised by humans, or synthesized by both humans and AI technologies. With the newly constructed ITT, we studied one of the canonical fluid-structure interaction problems, VIV of bluff bodies, using rigid cylinder forced vibration experiments. The study serves as a realization of the not-so-new idea of the scientific robotic researcher. It demonstrates that with a careful calibration of the inherent uncertainty of the experimental facility and a selection of the proper machine learning tools (in the current research, basis and kernel functions of the GPR for QoIs), the ITT is capable of (a) adaptively and intelligently designing and conducting sequential experiments to study targeted QoIs (in the current research, hydrodynamic coefficients of a forced vibrating cylinder in uniform flow); (b) revealing the complex physics of the non-linear system with the same level of accuracy but a reduced number
of experiments by orders of magnitude compared to the traditional experimental sampling strategy; and (c) exploring a wider parametric space (in the current research, up to eight parameters) for new physical insights and scalings, which was infeasible in past research, and thus accelerates the scientific discovery.

One of the benefits for the GPR learning results not addressed in the current paper but that should be mentioned here, is that when the learning process stops, the ITT provides not only a collection of experiment data but also, more importantly, an accurate functional approximation of the targeted QoI. Such a functional representation opens new possibilities to use various optimization tools, while incorporating additional physical insights as constraints when applying the acquired data to predict or understand more complicated problems. For example, when predicting the VIV response of a long, slender marine riser in the ocean current using the hydrodynamic coefficients acquired from the rigid cylinder forced vibration experiment, currently, the method of searching data tables contained in databases, and employing parametric interpolation is used (40). The use of deep learning methods allows the application of effective optimization methods, expanding the parametric space to represent appropriate riser physical conditions, enabling the study of complex phenomena, such as the recently observed flexible cylinder VIV hysteretic response associated with mode switch (42), or extremely large vibrations observed in some experiments (43).

In the current active learning framework for adaptive sequential experimentations, shown in the flowchart of Fig. 1, we selected GPR as the main learning tool. As we have shown, the performance of the GPR learning, such as the convergence rate, depends heavily on the choice of the kernel functions. In our current study, the candidates are drawn from several standard kernel functions of the Matern family. Other models, such as the newly developed neural-net-induced Gaussian process (44), should be tested to improve the GPR learning performance, especially for rare events. More importantly, the selection of the “best” kernel of each QoI in the current research is merely via brute-force by comparing the performance of the converged GPR learning results with different kernels of the same experiment. Several recent studies (45, 46) on learning the kernel from the data could be exploited so that in the sequential experiment, not only the prediction of the QoIs, but also the kernels, are updated. With an increasing number of experiments, a “better” form of the kernel will emerge to represent the experimental data, which may accelerate the learning process, but more interestingly, may better reveal or interpret the hidden physics of the data by examining the learned kernel form.

One limitation using GPR is that the computational expense quickly increases with the number of the experiment (47). This limits the total experiment number and the dimension for the input parametric space. Deep neural networks (NNs) are known for their high expressivity and the ability to handle large dimensions of input parameters and big data (48). Hence, the next generation of the ITT should include a deep NN that can handle problems with hundreds of input parameters, unlike GPR. This will require robust methods of uncertainty quantification of NN (49), which is a subject of ongoing work (50).

The searching strategy is another key component in the active learning for the sequential experimentation. In the current study, we find the parameters of the next experiment input by merely applying the strategy to find the maximum of the standard deviation. In the future, prior knowledge of the problem (51), such as the well understood physics and/or the engineering requirements, can be incorporated to form acquisition functions with multiple objectives and/or additional physical-informed constraints (52).
It should be noted that although the robotic part of the apparatus we describe herein is relatively simple, consisting of automatically conducting forced vibration experiments with prescribed motions, our laboratory has pioneered and made available the use of a more complex apparatus for fluid mechanics research involving simultaneous experimental testing of models connected with virtual systems through real-time simulation (53, 54), which we can use within the same scheme. Our methodology has been emulated in other laboratories, denoted as cyber-testing (55-57), enabling complex system representation in the laboratory. Hence, the same procedure is applicable with complex cyber-physical systems that constitute an elaborate robotic apparatus.

We should add that the machine learning methodology in this paper is not limited to fluid mechanics, and can be easily transferred to other areas, e.g., in experimental solid mechanics, where a large number of specimens is required to quantify the modulus of elasticity, the yield stress, and the onset of fracture. Hence, combined with advanced manufacturing technologies (58-60) that are capable of generating versatile prototypes in a short amount of time, we foresee a great potential of automatic sequential experimentation to map material and structural properties (61), to obtain understanding that may lead to new advances, such as developing the next-generation of morphing wings for aviation (62). Similarly, this methodology is readily applicable to non-destructive evaluation of materials, where uncertainty quantification and automation will accelerate considerably such testing. Furthermore, in an application outside the well-controlled environment of the laboratory, the methodology can be used with multiple, inexpensive robots to form dynamic swarms (63, 64) that will enable adaptive and swift monitoring and exploration of the environment (65).

In conclusion, Robotic Scientists should be playing a greater role in the automation of science, in particular in engineering, where there are many opportunities to implement machine learning methodologies analogous to the one we presented here.

**Materials and Method**

In this section, we first review GPR, the learning algorithm selected in the current research. Then we define the hydrodynamic coefficients of the rigid cylinder in the forced vibration experiments conducted, followed by the review of Morris method for global sensitivity analysis.

**Gaussian process regression** is a non-parametric method of modeling unknown functions from a finite set of training points to make predictions, and it has been successfully applied in various field to explore the hidden physics from the data (65-69). Furthermore, GPR provides quantification of the uncertainty based on the selected kernel function and the estimated measurement uncertainty, which can guide the sequential experimentation adaptively to explore the parametric space and predict QoIs. We briefly review GPR as it is an essential component of ITT’s algorithm, but we refer to the book by Rasmussen and Williams (34) for a detailed exposition.

The experiment dataset \( \mathcal{D} = \{ (\mathbf{x}_i, y_i) | i = 1, \ldots, n \} \) consists of \( n \) observations of the output \( y \) with the input \( \mathbf{x} \in \mathbb{R}^s \) of \( s \) dimensions. Let us consider the following map between the inputs and output,

\[
y(\mathbf{x}) = \mathbf{h}(\mathbf{x})^T \boldsymbol{\beta} + f(\mathbf{x}) + \varepsilon,
\]

(4)
where $h(x)$ are a set of fixed basis functions that transform the input $x \in \mathbb{R}^d$ into new vectors of $h(x) \in \mathbb{R}^q$, and $\beta$ is a $q$-by-1 vector of basis function coefficients. $f(x) \sim \text{GP}(0, k(x, x'))$ is the bias of a zero mean GP with the covariance (kernel) function $k(x, x')$ parameterized by a set of hyperparameters $\mathcal{G}$, and thus $k(x, x'|\mathcal{G})$, and the measurement noise $\varepsilon \sim N(0, \sigma^2)$ is assumed independent between every two outputs. Therefore, the probability distribution of outputs $y$ of $n$ observations given $X$ (matrix of size $r \times n$) and $f$ (vector of size $n \times 1$) can be modeled as follows,

$$P(y|X, f) \sim N(H\beta + f, \sigma^2 I),$$

(5)

where

$$X = \begin{pmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{pmatrix}, y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, H = \begin{pmatrix} h(x_1)^T \\ h(x_2)^T \\ \vdots \\ h(x_n)^T \end{pmatrix}, f = \begin{pmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \end{pmatrix}.$$ (6)

The joint distribution of $f = (f(x_1), f(x_2), \ldots, f(x_n))^T$ given $X$ is

$$P(f|X) \sim N(0, K(X, X)),$$ (7)

where $K(X, X)$ is the covariance matrix of the following form,

$$K(X, X) = \begin{pmatrix} k(x_1, x_1) & k(x_1, x_2) & \cdots & k(x_1, x_n) \\ k(x_2, x_1) & k(x_2, x_2) & \cdots & k(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ k(x_n, x_1) & k(x_n, x_2) & \cdots & k(x_n, x_n) \end{pmatrix}.$$ (8)

To estimate the parameters $\beta$, $\mathcal{G}$ and $\sigma^2$, given the dataset $\mathcal{D}$, we maximize the marginal likelihood $P(y|X) = P(y|f, X)P(f|X) \sim N(H\beta, \sigma^2 I + K(X, X|\mathcal{G}))$ as a function of $\beta$, $\mathcal{G}$ and $\sigma^2$. Therefore, the best estimate $\hat{\beta}$, $\hat{\mathcal{G}}$ and $\hat{\sigma^2}$ take the form,

$$\hat{\beta}, \hat{\mathcal{G}}, \hat{\sigma^2} = \arg \min_{\beta, \mathcal{G}, \sigma^2} \left[-\log P(y|X)\right],$$ (9)

where the negative log marginal likelihood function is shown as follows,

$$-\log P(y|X) = \frac{1}{2}(y - H\beta)^T \left[ \sigma^2 I + K(X, X|\mathcal{G}) \right]^{-1} (y - H\beta) + \frac{n}{2} \log 2\pi$$

$$+ \frac{1}{2} \log \left| \sigma^2 I + K(X, X|\mathcal{G}) \right|.$$ (10)
With the best estimate of $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\sigma}^2$, given the dataset $\mathcal{D}$, we make the prediction on the distribution of the output $y^*$ with the input $x^*$ as follows,

$$P(y^*|y,X,x^*) = \frac{P(y^*,y|X,x^*)}{P(y|X,x^*)} \sim N(\bar{y}^*,\sigma^*(x^*)),$$

where

$$\bar{y}^* = h(x)^T \hat{\beta} + K(x^*,X|\hat{\gamma}) \left[ \hat{\sigma}^2 I + K(X,X|\hat{\gamma}) \right]^{-1} (y - H \hat{\beta}),$$

$$\sigma^*(x^*)^2 = \hat{\sigma}^2 + k(x^*,x^*|\hat{\gamma}) - K(x^*,X|\hat{\gamma}) \left[ \hat{\sigma}^2 I + K(X,X|\hat{\gamma}) \right]^{-1} K(x^*,X|\hat{\gamma}).$$

Therefore, the parameters of the next experiment input $\hat{x}^*$ can be found by finding the maximum of $\sigma^*(x^*)$ as a function of $x^*$ as follows,

$$\hat{x}^* = \text{arg max}_{x^*} \left[ \sigma^*(x^*) \right].$$

**Using the rigid cylinder forced oscillation to study VIV:** In this section, we describe the three types of rigid cylinder forced vibration experiment performed in the current study and define the corresponding hydrodynamic coefficients.

**Cross-flow only forced vibration**

When a cylinder of length $l$ and diameter $d$ is forced to follow a sinusoidal trajectory in the cross-flow direction to the uniform inflow with velocity $U$ as follows,

$$Y(t) = Y_0 \cos(2\pi f_0 t),$$

where $Y_0$ and $f_0$ are the cross-flow oscillation amplitude and frequency, respectively. The lift and drag forces on the cylinder can be modeled as follows,

$$L(t) = L_0 \cos(2\pi f_0 t + \phi_0),$$

$$D(t) = D_m + D_0 \cos(4\pi f_0 t + \varphi_0),$$

where $D_m$ is the magnitude of the mean drag force, $L_0$ and $D_0$ are the magnitudes of the oscillating lift and drag forces at frequencies $f_0$ and $2f_0$ respectively, $\phi_0$ is the phase difference measured between the cross-flow motion and the oscillating lift force and $\varphi_0$ is the phase difference measured between the cross-flow motion and the oscillating drag force. Therefore, the hydrodynamic coefficients, i.e., mean drag coefficient $C_d$, lift coefficient in phase with velocity $C_l$, and added mass coefficient in the cross-flow direction $C_m$, are functions of non-dimensional cross-flow amplitude $A_y/d = Y_0/d$ and the reduced frequency $f_r = f_0 d/U$ as follows,
\[ C = C_0 \left( \frac{A_x}{d}, f_r \right). \]  

(16)

Therefore, from the experiments, we can measure the three hydrodynamic coefficients as follows,

\[ C_d = \frac{2D_m}{\rho l d U^2}, \]
\[ C_{lv} = \frac{2L_0 \sin(\phi_0)}{\rho l d U^2}, \]
\[ C_{my} = \frac{L_0 \cos(\phi_0)}{2\pi \nu Y_0 f_0}, \]

(17)

where \( \rho \) is the fluid density and \( \nu \) is the cylinder fluid displacement of \( \nu = \frac{\pi}{4} d^2 l \).

**Inline-and-cross-flow-combined forced vibration**

When the rigid cylinder is forced to oscillate harmonically in uniform inflow, its trajectory can be described as follows,

\[ Y(t) = Y_0 \cos(2\pi f_0 t), \]
\[ X(t) = X_0 \cos(4\pi f_0 t + \theta), \]

(18)

where \( X_0 \) is the oscillation amplitude in the inline direction and \( \theta \) is the phase difference imposed between the inline and cross-flow motions, where we define the trajectory as counter-clockwise when \( \theta \in [0, \pi] \) and clockwise when \( \theta \in [\pi, 2\pi] \). The lift and drag forces on the cylinder can be again modeled as in equation (2). The hydrodynamic coefficients are hence functions of four parameters as follows,

\[ C = C_1 \left( \frac{A_x}{d}, \frac{A_y}{d}, V_r, \theta \right), \]

(19)

where \( A_x/d = X_0/d \) is the non-dimensional inline amplitude, \( V_r = 1/f_r \) is the reduced velocity, inverse of the reduced frequency. The targeted QoI in the current paper, \( C_{iv} \) of rigid cylinder inline-and-cross-flow-combined forced vibration is the same as in the equation (4).

**Inline-and-cross-flow-combined forced vibration with double frequencies**

Instead of single frequency vibration, when vibrating simultaneously at two frequencies, the cylinder trajectory is prescribed as follows,

\[ Y(t) = Y_0 \cos(2\pi f_0 t) + Y_2 \cos(2\pi f_2 t), \]
\[ X(t) = X_0 \cos(4\pi f_0 t + \theta) + X_2 \cos(4\pi f_2 t + \theta_2), \]

(20)
where $X_2$, $Y_2$ and $\theta_2$ are the inline, cross-flow amplitudes and phase difference of the second vibration frequency $f_2$. Therefore, the lift and drag forces on the cylinder can be modeled as

$$L(t) = L_0 \cos(2\pi f_0 t + \phi_0) + L_2 \cos(2\pi f_2 t + \phi_2),$$

$$D(t) = D_0 + D_2 \cos(4\pi f_2 t + \phi_2),$$

where $L_2$, $D_2$ are the magnitudes of the lift and drag forces associated with second frequency $f_2$, and $\phi_2$, $\phi_2$ are the phases between forces and motions of second frequency $f_2$. The targeted QoI in the current paper, $(C_{iv})_{\text{double}}$, is associated with the first vibration frequency $f_0$, and it is a function of eight parameters, i.e.,

$$(C_{iv})_{\text{double}} = C_2 \left[ \left( \frac{A_x}{d}, \frac{\theta_1}{d}, V_r, \theta \right), \left( \frac{A_{s2}}{d}, \frac{A_{p2}}{d}, V_{s2}, \theta_2 \right) \right],$$

which can be calculated from the experiment as follows,

$$(C_{iv})_{\text{double}} = \frac{2L_0 \sin(\phi_0)}{\rho d U^2}. \quad (23)$$

**Morris method for global sensitivity analysis** is widely used to screen the important input parameters for a given model or problem (41).

Given a normalized input space $S = [0,1]$ with a $s$-dimensional, $p$-level full factorial grid, that is $x_i \in \{0,1/(p-1),2/(p-1),\ldots,1\}$ for $i = 1,\ldots,s$, for a given value of $x \in S$, the elementary effect of $x_i$ can be calculated as follows,

$$\delta_i(x) = \frac{y(x_{1},x_{2},\ldots,x_{i-1},x_{i}+\Delta,x_{i+1},\ldots,x_{s})-y(x)}{\Delta}, \quad (25)$$

where $\Delta$ is a predetermined multiple of $1/(p-1)$, and therefore $x_i \leq 1-\Delta$. Given the output $y$ with a screening plan $X$, the sample mean and standard deviation of a set of $\delta_i(x)$ values can be estimated for each input parameter.

The screening plan $X$ is built from the sampling matrix $B$, where $B$ is a $(s+1)\times s$ matrix of 0’s and 1’s with the key property that for every column $i = 1,\ldots,s$, there are two rows of $B$ differ only in their $i^{th}$ entries. We denote $\tilde{B}$ is the random orientation of $B$, and it can be expressed as follows,

$$\tilde{B} = (J_{s+1,1}\tilde{x} +(\Delta/2)\left[ (2B-J_{s+1,s}) \right] \tilde{D} + J_{s+1,s}) \tilde{P},$$

where $J_{l,m}$ is $l$-by-$m$ matrix of 1’s, $\tilde{D}$ is a $s$-dimensional diagonal matrix where each element on the diagonal is either +1 or -1 with equal probability, $\tilde{x}$ is a randomly selected point in $s$-dimensional, $p$-level discretized input space $S$, $\tilde{P}$ is a $s$-by-$s$ random permutation matrix where
each column contains only one element of 1 and all other equal to 0 and no two columns has 1’s in the same position. Therefore to evaluate \( r \) elementary effects for each parameter, the screening plan \( X \) is constructed from \( r \) random orientations as follows,

\[
X = \begin{bmatrix}
\tilde{B}_1 \\
\tilde{B}_2 \\
\vdots \\
\tilde{B}_r
\end{bmatrix}.
\]

(27)

References and Notes

6. J. Wise, These robots are learning to conduct their own science experiments. Bloomberg Businessweek. (April 11, 2018).
43. K. Raghavan, M. M. Bernitsas, Experimental investigation of Reynolds number effect on vortex induced vibration of rigid circular cylinder on elastic supports. *Ocean Eng.* **38** (5-6), 719-731 (2011).


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Figures and Tables

Table 1. Selected basis and kernel functions for different QoIs.

<table>
<thead>
<tr>
<th></th>
<th>$C_d$</th>
<th>$C_{lv}$</th>
<th>$C_{my}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>basis</td>
<td>Pure Quadratic</td>
<td>Linear</td>
<td>Pure Quadratic</td>
</tr>
<tr>
<td>kernel</td>
<td>ARD Matern 3/2</td>
<td>ARD Matern 5/2</td>
<td>ARD Matern 5/2</td>
</tr>
</tbody>
</table>
Fig. 1. Schematic image of the ITT with the key steps for sequential learning of complex fluid-structure dynamics. The image of the ITT (A) shows the experimental model consisting of a cylinder and sensors mounted on the main carriage, which can be driven to perform combined in-line, cross-flow, and rotational motions. The graphic user interface (GUI) of the ITT controller, recording motion and force signals, is shown at the bottom right of the figure. The process of the ITT commences once a hypothesis is proposed (such a hypothesis is human generated, or, in the future, may be synthesized in coordination by robots, computers and humans). Then the ITT performs the adaptive sequential experiment to learn target QoIs, interrupted only by pause periods between experiments to avoid cross-contamination of the results between successive experiments. Upon convergence, the results of learned QoIs are further post-processed to examine the validity of the hypothesis. During sequential experimental testing there is no human-in-the-loop. B provides an overview of the ITT with main components of a 10m tank, a carriage of 3-axis robotic linear stage, a computer and motor controllers.
Fig. 2. A demonstration of GPR learning sequence for $C_{lv}$ of a rigid cylinder forced vibration in uniform flow at $Re = 12,000$. A.1 - A.5 show contours of the mean of the predicted $C_{lv}$ versus reduced frequency $f_r$ (x-axis) and non-dimensional vibration amplitude $A_y/d$ (y-axis) along with the standard deviation plot (inset). The black dots in each contour denote the existing data used for GPR learning at the current iteration. The black square denotes the new experiment performed for the current iteration while the red star represents the next experiment guided by the $\sigma_{\text{max}}$ in the standard deviation. B. Plot of the maximum standard deviation $\sigma_{\text{max}}$ versus experiment number. The horizontal dashed line corresponds to $3\sigma$, where $\sigma$ is the standard deviation for a reference case as described in the text (see also Table S1 and Fig. S1).
Fig. 3. Investigation of the GPR learning convergence for different types of kernels. The plots show $\sigma_{\text{max}}$ of each iteration with different kernel functions and a fixed basis function for: A. $C_d$, B. $C_v$ and C. $C_{nv}$. The horizontal dash line corresponds to $3\sigma_r$, where $\sigma_r$ is the standard deviation for a reference level as described in the text (see also Table S1. and Fig. S1).
Fig. 4. GPR vs. uniform sampling. Comparison of adaptive (A.1, B.1, C.1) GPR learning (multi-output) versus uniform (lattice) sampling (A.2, B.2, C.2). Contours of $C_d$ (A1 - 75 experiments; A2 - 2,268). Contours of $C_h$ (B1 - 77 experiments; B2 - 2,268 experiments). Contours of $C_m$ (C1 - 90 experiments; C2 - 2,268 experiments). A.3 - C.3 plots of the comparison of the average value of 30 randomly selected points (blue dots in A.2.) between uniform sampling (red dashed line) and GPR learning (black dashed line) as a function of the experiment number. The blue shaded region denotes the two standard deviation margin (averaged over the 30 selected points) as a function of the experiment number.
Fig. 5. Reynolds number effect in three-dimensional parametric space. Converged hydrodynamic coefficients in the three-dimensional parametric space ($f_r, A_r/d, Re$) using GPR learning (multi-output) strategy. Iso-surfaces of the hydrodynamic coefficients: A. $C_d$ of 207 experiments (blue surface: $C_d = 1.3$; black surface: $C_d = 2$; red surface: $C_d = 3$; green surface: $C_d = 4$); B. $C_{lv}$ of 1,036 experiments (green surface: $C_{lv} = -3$; blue surface: $C_{lv} = -1$; black surface: $C_{lv} = 0$; red surface: $C_{lv} = 0.3$); C. $C_{my}$ of 1,288 experiments (black surface: $C_{my} = 0$; red surface: $C_{my} = 2$).
Fig. 6. Exploration of large parametric space (in this example: 8 parameters). Comparison of $C_{iv}$ for a rigid cylinder undergoing combined in-line and cross-flow forced vibrations in uniform flow at $Re = 5,715$ obtained in single frequency (involving 4 parameters, total 755 experiments) and in two frequencies (involving 8 parameters, total 3,944 experiments) experiments. A shows contours of $C_{iv}$ versus $V_x$ and $\theta/\pi$ for experiments of single frequency at fixed $A_x/d = 0.15$ and $A_y/d = 0.75$. B.1 shows contours of $C_{iv}$ versus $V_x$ and $\theta/\pi$ for experiments of double frequency at $A_x/d = 0.15$ and $A_y/d = 0.75$, same as in A, and fixed second frequency component of $A_{z2}/d = 0.34$, $A_{z2}/d = 0.14$, $V_{z2} = 11.75$ and $\theta_{z2}/\pi = 1.5$. B.2 - B.5 show contours of $C_{iv}$ versus $f$, and $\theta/\pi$ with only one fixed input changed; compare to B.1: B.2, $A_{z2}/d = 0.93$; B.2, $A_{z2}/d = 0.25$; B.3, $V_{z2} = 5.25$; B.4, $\theta_{z2}/\pi = 0.5$. The results of the sensitivity analysis on $\chi$ are shown in C. The sensitivity measures $(|\mu_\chi|, |\sigma_\chi|)$ for each parameter have been normalized by the value of the highest sensitivity measure $(|\mu_\chi^\text{max}|, |\sigma_\chi^\text{max}|)$ of the most sensitive parameter $V_{z2}$. 
**SUPPLEMENTARY MATERIALS**

**Table**

Table S1. Statistics of hydrodynamic coefficients of a stationary rigid cylinder in uniform flow at $Re = 12,000$.

<table>
<thead>
<tr>
<th></th>
<th>$C_d$</th>
<th>$C_l^{mean}$</th>
<th>$C_l^{rms}$</th>
<th>$St$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>1.1980</td>
<td>0.3959</td>
<td>0.1173</td>
<td>0.1931</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.0398</td>
<td>0.0617</td>
<td>0.0098</td>
<td>0.0025</td>
</tr>
</tbody>
</table>
Fig. S1. Histogram of $C_d$ of a stationary rigid cylinder in uniform flow at $Re = 12,000$. The standard deviation measured for $C_d$ is selected as the reference value for convergence $\sigma$. 
Fig. S2. Evolution of GPR learning sequence for $C_r$ of a rigid cylinder forced vibration in uniform flow at $Re = 12,000$. In this learning, "Linear" basis function and "ARD Matern 3/2" kernel function is selected.
Fig. S3. $C_{lv}$ of a rigid cylinder forced vibration from GPR learning at various $Re$ number values. A: $Re = 2,000$; B: $Re = 7,000$; C: $Re = 12,000$; D: $Re = 17,000$. 
Fig. S4. $C_d$ of a rigid cylinder forced vibration from GPR learning at various $Re$ number values. A: $Re = 2,000$; B: $Re = 7,000$; C: $Re = 12,000$; D: $Re = 17,000$. 
Fig. S5. $C_{wy}$ of a rigid cylinder forced vibration from GPR learning at various $Re$ number values. A: $Re = 2,000$; B: $Re = 7,000$; C: $Re = 12,000$; D: $Re = 17,000$. 
Data file

All the data can be downloaded from the following link:
https://www.dropbox.com/sh/16df5cqu2c7v94q/AABABCux6G7rnYOI9g3H-Krra?dl=0

Data file S1. Sequential experimental data for hydrodynamic coefficients of a cross-flow only vibrating rigid cylinder at $Re = 12,000$.

Data Explanation:
1. Initial (datatype: structure): initial sparse experiments
   a. 1st column: $A_j/d$;
   b. 2nd column: $f_r$;
   c. 3rd column: $C_d$;
   d. 4th column: $C_b$;
   e. 5th column: $C_{ny}$.
2. SequentialExp (datatype: cell): sequential experiments
   a. Sample_with_Cd (datatype: structure): the experiment conducted based on $C_d$;
   b. Sample_with_Clv (structure): the experiment conducted based on $C_b$;
   c. Sample_with_Cmy (structure): the experiment conducted based on $C_{ny}$.

Data file S2. Sequential experimental data for hydrodynamic coefficients of a cross-flow only vibrating rigid cylinder at various $Re$ from 1,200 to 19,000.

Data Explanation:
1. Initial (datatype: structure): initial sparse experiments
   a. 1st column: $A_j/d$;
   b. 2nd column: $f_r$;
   c. 3rd column: $Re$;
   d. 4th column: $C_d$;
   e. 5th column: $C_b$;
   f. 6th column: $C_{ny}$.
2. SequentialExp (datatype: cell): sequential experiments
   a. Sample_with_Cd (datatype: structure): the experiment conducted based on $C_d$;
   b. Sample_with_Clv (structure): the experiment conducted based on $C_b$;
   c. Sample_with_Cmy (structure): the experiment conducted based on $C_{ny}$.

Data file S3. Sequential experimental data for hydrodynamic coefficients of a cross-flow and in-line combined single-frequency vibrating rigid cylinder at $Re = 5,715$.

Data Explanation:
1. Initial (datatype: structure): initial sparse experiments
   a. 1st column: $A_j/d$;
   b. 2nd column: $A_j/d$;
   c. 3rd column: $V_r$;
   d. 4th column: $\theta$;
   e. 5th column: $C_b$.
2. SequentialExp (datatype: cell): sequential experiments
   a. Sample_with_Clv (structure): the experiment conducted based on $C_b$. 
Data file S4. Sequential experimental data for hydrodynamic coefficients of a cross-flow and in-line combined double-frequency vibrating rigid cylinder at \( Re = 5,715 \).

Data Explanation:
1. \textit{Initial} (datatype: structure): initial sparse experiments
   a. 1\(^{st}\) column: \( A_y/d \);
   b. 2\(^{nd}\) column: \( A_x/d \);
   c. 3\(^{rd}\) column: \( V_r \);
   d. 4\(^{th}\) column: \( \theta \);
   e. 5\(^{th}\) column: \( A_{y2}/d \);
   f. 6\(^{th}\) column: \( A_{x2}/d \);
   g. 7\(^{th}\) column: \( V_{r2} \);
   h. 8\(^{th}\) column: \( \theta_2 \);
   i. 9\(^{th}\) column: \( C_{h_r} \).
2. \textit{SequentialExp} (datatype: cell): sequential experiments
   a. Sample\_with\_C\_lv (structure): the experiment conducted based on \( C_{h_r} \).
Movie
Movie S1. Experimental process of ITT sequential learning on $C_{ir}$ of a cross-flow only vibrating rigid cylinder at $Re = 12,000$.

Movie S2 ITT sequential experiment of $Re$ effect on a cross-flow only vibrating rigid cylinder.